The Slope-Intercept Form of a Line

For GSP5

The slope-intercept form of a line, $y = mx + b$, is one of the best-known formulas in algebra. In this activity you'll learn about this equation first by exploring one line, and then by exploring whole *families* of lines.

SKETCH AND INVESTIGATE

You'll start this activity with $m = 2$ and $b = 1$ as you explore the line $y = 2x + 1$.

Choose **Graph | Define Coordinate System.** To hide the points, select them and choose **Display | Hide Points.**

1. In a new sketch, define a coordinate system and hide the points (0, 0) and (1, 0).

Q1 For $y = 2x + 1$, what is y when $x = 0$? Write your answer as an ordered pair.

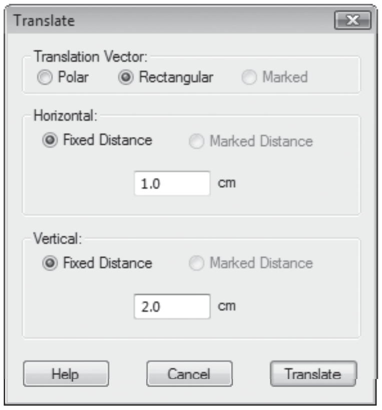
Choose **Graph | Plot Points.** Enter the coordinates in the Plot Points dialog box, click **Plot;** then click **Done.**

2. Plot this point. Why does it make sense to call this point the *y-intercept*?

Q2 You found that the y-intercept of $y = 2x + 1$ is 1. What is the y-intercept of $y = 3x + 7$? Explain why the y-intercept of $y = mx + b$ is always b.

You've learned that *slope* can be written as *rise/run*. The slope of the line $y = 2x + 1$ is 2, which you can think of as 2/1 (*rise* = 2 and *run* = 1).

3. Translate your plotted point using this slope. Choose **Transform | Translate,** use a rectangular translation vector, and enter 1 for the run (horizontal) and 2 for the rise (vertical).



To measure the coordinates, choose **Measure | Coordinates.**

Q3 What are the coordinates of the new point? Substitute them into $y = 2x + 1$ to show they satisfy the equation.

Q4 Translate the new point by the same *rise* and *run* values to get a third point. Find the coordinates of this third point, and verify that it satisfies the equation $y = 2x + 1$.

4. Select any two of the three points you've plotted, and choose **Construct | Line.**

What you've done so far is one technique for plotting lines in the form $y = mx + b$:

- Plot the y-intercept $(0, b)$.
- Rewrite m as *rise/run* (if necessary).
- Find a second point by translating the y-intercept by *rise* and *run.*
- Connect the points to get the line. Plot a third point to check the line.

If m is a decimal such as 1.5, write it as a fraction such as 3/2. If it's a whole number such as 3, write it as a fraction such as 3/1.

Q5 Using the method just described, plot these lines on graph paper.

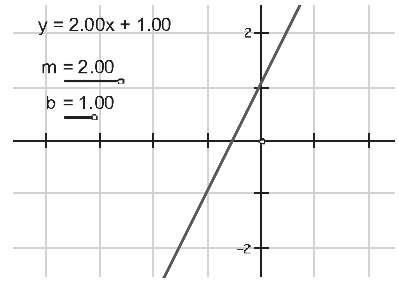
a. $y = 3x - 2$ b. $y = (2/3)x + 2$

c. $y = -2x + 1$ d. $y = 2.5x - 3$

EXPLORING FAMILIES OF LINES

Now that you've plotted a line, focus on how m and b affect the equation.

5. Open **Slope Intercept.gsp.**



The graph of $y = 2x + 1$ is already plotted. You can change m and b by adjusting their sliders.

To adjust a slider, drag the point at its tip.

Q6 Adjust slider m and observe the effect. Describe the differences between lines with $m > 0$, $m < 0$, and $m = 0$. What happens to the line as m becomes increasingly positive? Increasingly negative?

Q7 Now adjust slider b. Describe the effect this value has on the line.

6. Select the line and choose **Display | Trace Line.**

Q8 Adjust m and observe the trace pattern that forms. Describe the lines that appear when you change m. What do they have in common?

To erase traces left by the line, choose **Display | Erase Traces.**

Q9 Erase the traces and adjust b. How would you describe the lines that form when you change b? What do they have in common?

7. Turn off tracing by selecting the line and choosing **Display | Trace Line** again. Erase any remaining traces.

Q10 For each description below, write the equation in slope-intercept form. To check your equation, adjust m and b so that the line appears on the screen.

a. slope is 2.0; y-intercept is $(0, -3)$

b. slope is -1.5; y-intercept is $(0, 4)$

c. slope is 3.0; x-intercept is $(-2, 0)$

d. slope is -0.4; contains the point $(-6, 2)$

e. contains the points $(3, 5)$ and $(-1, 3)$

EXPLORE MORE

Q11 Attempt to construct a line through the points $(3, 0)$ and $(3, -3)$ by adjusting the sliders in the sketch. Explain why this is impossible. (Why can't you write its equation in slope-intercept form?)

Q12 Can you construct the same line with two different slider configurations? If so, provide two different equations for the same line. If not, explain why.